Kingdom of Saudi Arabia
Ministry of Higher Education
Majmaah University Faculty of Science-Zulfi
Department of Computer Science & Information

جامعة المجمعة
Majmaah University

Advanced Mathematics For CS

"Discrete Mathematics"

Prepared By:
Dr. Eng. Moustafa Reda AbdALLAH

1st Term : 1435 - 1436 h

Course goals

- Mathematical Reasoning المنطق الرياضي/السببية
  - Logic, inference/deduction/conclusion حدس, proof

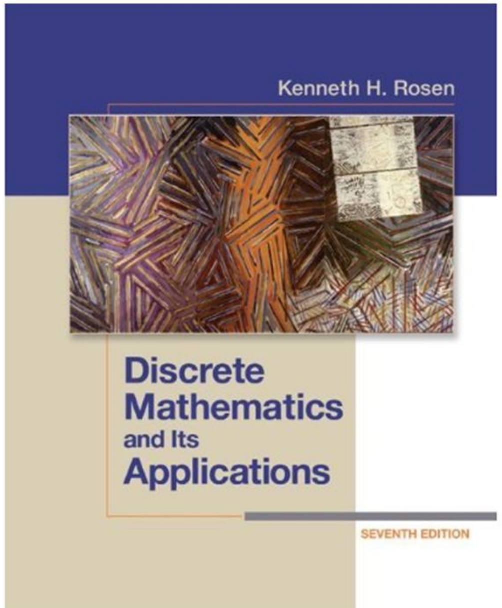
- Graph Theory Applications تطبيقات نظرية المخططات
  - Euler's and Hamilton's Paths and Circuits, Shortest Path, Trees, Huffman's Code.

Main Topics

1. Propositional Logic.
2. Set Theory.
3. Sequences & Theory of Proofs.
4. Graph Theory and its Applications.

Textbook

- *Discrete Mathematics and Its Applications* ; Kenneth H. Rosen, 7^{th} edition, McGraw Hill, 2012.



Recommended Books

- ***A Beginner's Guide to Discrete Mathematics;*** **W. D. Wallis; Birkhäuser; 2003.**

- ***Discrete Mathematics;*** Richard Johnsonbaugh; 7th edition; Prentice Hall International, 2009.

- ***Thinking Mathematically;*** Robert Blitzer; 4th edition; Pearson Prentice Hall; 2010.

- ***Mathematical Ideas;*** Charles D. Miller, Vern E. Heern, John Hornsby; Expanded 10th edition; Pearson Addison Wesley; 2004.

Prerequisites

- Basic knowledge of calculus .
- Basic knowledge in computer science.

Grading توزيع الدرجات

- 1^{st} Midterm (6^{th}/7^{th} Week): 20 Points.
- 2^{nd} Midterm (12^{th}/13^{th} Week): 20 Points.
- Activities: 20 Points.
- Final Exam: 40 Points.

- N.B.

Activities:

- Class Attendance,
- Assignments & Homework,
- Quizzes & Net Researches.
- Class Participation, Presentations (weekly).

Class Policy

- Never use smart phones in class.
- All the lectures and notes will be available for the students.
- Weekly homework assigned on 1st part of the lecture and due (must be achieved يجب أن ينجز) before the 2nd part of the same weekly lecture.
- Must be your own work.
- Homework returned in class or by e-mail.

Lecture One

1.1 Logic Concepts

The logic mimic يقلد /imitate/copy our intuitions حدس/feelings/insights by setting down (explaining) constructs/concepts/ideas/theories/hypotheses that behave analogously بصورة متشابهة/مناظرة.

The logic furnishes statements that describe the surrounding world that can be true/false. That the world is made up of objects and that objects can be organized to form collections of statements.

Used in numerous applications: circuit design, programs, verification of correctness of programs, artificial intelligence, etc.

1.2 Propositional Logic المنطق الخبري/التصريحي

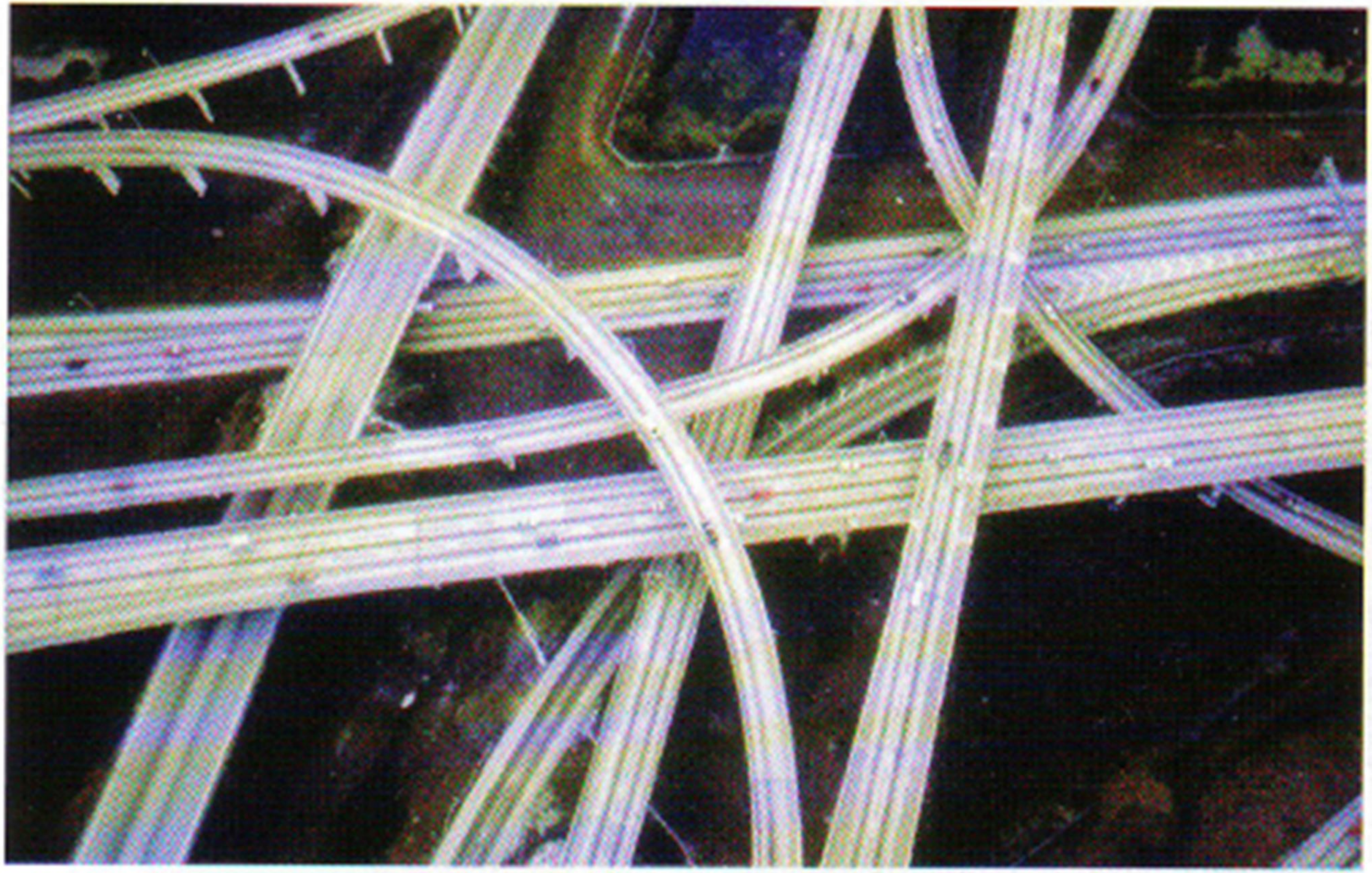
- Is a declarative statement جملة تصريحية/خبرية that is either true or false, but not both. It is not an Opinion, a Command, or a Question. Such as:
  - Washington, D.C., is the capital of USA
  - California is adjacent to New York
  - 1+1=2
  - 2+2=5

- Not declarative
  - What time is it? → Question
  - Read this carefully → Command
  - It is a nice day → Opinion

1.2 Propositional Logic
المنطق الخبري/التصريحي

- Elementary/Simple statement is atomic, with a verb, subject and object but no connectives (NOT, OR, AND, If…Then, Iff)
- Construct correct mathematical arguments الحجج.
- Give precise meaning to mathematical statements
- Focuses on the relationships among statements.
- Rules are used to distinguish between valid (True) and invalid (False) arguments/statement. Hence logic assures us for any combination/connection of these statements as being True/False.

1.3 False/True Statements



1.3 False, True, Statements

Axiom مسلمة: ***False*** is the opposite to ***Truth***.

A statement/declaration/declarative-statement is a description of something (sentence), which **Is Not** an **Opinion**, a **Command**, or **Question**. A statement Is A Sentence عبارة That Is Either True Or False, but not both simultaneously.

Examples of statements:

- **I'm 58 years old.**
- **I have 888 children.**
- **I always tell the truth.**
- **I'm lying to you.**

Q's: Which statements are:

True? **False**? **Both**? **Neither**?

1.3 False, True, Statements

True: I'm 58 years old.

False: I have 888 children.

I always tell the truth.

Both: IMPOSSIBLE, by our Axiom.

1.3 False, True, Statements

Neither:

I'm lying to you. (If viewed on its own من تلقاء نفسها)

Suppose that *S* = "I'm lying to you." were **true**. In particular, I am actually lying, so *S* is **false**. So it's both **true and false**, impossible by the Axiom.

الجملة S صادقة: أنا أكذب في كلامي. و من ثم عندما أخبرك (في الجملة S) أنني أكذب عليك: فأنني أكذب في كذبي عليك. أي أننيفعلاً لا أكذب عليك أي أن **الجملة S كاذبة** .

Okay, so I guess *S* must be **false**. But then I must not be lying to you. So the statement is **true**. Again it's both **true and false**.

الجملة S كاذبة: أنا لا أكذب في كلامي. و من ثم عندما أخبرك (في الجملة S) أنني أكذب عليك فأنني فعلاً أكذب عليك أي أن **الجملة S صادقة** .

In both cases we get the opposite of our assumption, so *S* is neither true nor false.

Exercise Sets 1.1 - 1.3

I. In Exercises 1- 12, determine whether or not each sentence is a Statement. Specify which is True and which is False.

1. George W. Bush was the Democratic candidate for president in 2004.
2. John Kerry was the Republican candidate for president in 2004.
3. Take the most interesting classes you can find.
4. Don't try to study on a Friday night in the dorms.
5. The average human brain contains 100 billion neurons.
6. There are 2.500.000 rivets in the Eiffel Tower.
7. Is the unexamined life worth living?
8. Is this the best of all possible worlds?
9. Some Catholic countries have legalized same-sex marriage.
10. Some U.S. presidents were assassinated.
11. $9 + 6 = 16$
12. $9 \times 6 = 64$

Exercise Sets 1.1 - 1.3

II. Decide whether each of the following is a statement or is not a statement. Specify which is True and which is False.

1. $5 + 8 = 12$ or $4 - 3 = 2$
2. Some numbers are negative.
3. Andrew Johnson was president of the United States in 1867.
4. Accidents are the main cause of deaths of children under the age of 8.
5. *Star Wars: Episode I—The Phantom Menace* was the top-grossing movie of 1999.
6. Where are you going today?
7. Behave yourself and sit down.
8. Kevin "Catfish" McCarthy once took a prolonged continuous shower for 340 hours, 40 minutes.
9. One gallon of milk weighs more than 4 pounds.

1.4 Symbolic Logic

المنطق الرمزي / ترميز المنطق

- In symbolic logic, we use lower case letters such as p, q, r, and s to represent statements. Here are two examples:
- p: Riyadh is the capital of KSA.
- q: Abin Al-Haythim is one of the greatest muslim scientists.
- <u>Hence</u>:
  - The letter p represents the 1st statement.
  - The letter q represents the 2nd statement.

1.5 Logical Operators/Connectives العوامل/الروابط المنطقية

- If you're wealthy or well educated, then you'll be happy.

- We can break this statement down into three basic sentences: You're wealthy, You're well educated, You'll be happy. These sentences are called simple statements because each one conveys/takes/sends/carries/transfers/delivers one idea with no connecting words. Statements formed by combining two or more simple statements are called compound statements. Logical connectives are used to join simple statements to form a compound statement. These Connectives include the words AND, OR, IF . . . THEN, and IF AND ONLY IF.
- Compound statements appear throughout written and spoken language. We need to be able to understand the logic of such statements to analyze information objectively. Starting from 1.10, we will concentrate our analysis on four kinds of compound statements.

1.5 Logical Operators/Connectives العوامل/الروابط المنطقية

- Are used to form لتكوين compound propositions تصريحات مركبة from simple ones.

- Negation (NOT, ~ , ¬) – Unary : Operates on one Statement. $\neg p$

- Conjunction الاقتران(AND, ∧) – Binary : Operates on two Statement. $p \wedge q$

- Disjunction الانفصال / Inclusive شامل OR (OR , v)
  Binary : Operates on two Statement. $p \vee q$

- Conditional/Implication شرط/مشاركة/تضمين/تورط/إستلزام (If…Then … , … → …)
  Binary : Operates on two Statement. $p \rightarrow q$

- Biconditional-statement/Equivellence التكافؤ (… If And Only If… , … ↔ …)
  Binary : Operates on two Statement. $p \leftrightarrow q$

1.5 Table of Logical Operators/Connectives
العوامل/الروابط المنطقية

<table>
<tr><th>Operation</th><th>Operator's Symbol</th><th>Usage Name</th><th>Java Form</th></tr>
<tr><td>Negation
النفي</td><td>¬</td><td>NOT</td><td>!</td></tr>
<tr><td>Conjunction
الوصل/الإقتران</td><td>∧</td><td>AND</td><td>&&</td></tr>
<tr><td>Disjunction
الفصل</td><td>∨</td><td>OR</td><td>||</td></tr>
<tr><td>Conditional
الشرط</td><td>→</td><td>If, Then</td><td>p ? q : r
If p Then q Else r</td></tr>
<tr><td>Biconditional
ثنائية الشرط</td><td>↔</td><td>Iff</td><td>(p && q)||(!p && !q)</td></tr>
</table>

1.6 Truth Tables

- Are the tables that indicate truth /falsity of a given proposition either in simple or complicated forms.

- True $\rightarrow$ T , 1
- False $\rightarrow$ F , 0

1.7 Negation

NOT p / ~p / ¬p

It is a Unary Operator (takes/affects One Argument)

TABLE 1 The Truth Table for the Negation of a Proposition.

p	$\neg p$
T	F
F	T

1.7 Negation

- "Today is Friday" ⇒ p
  - It is not the case that today is Friday ⇒ ¬p
  - Today is not Friday ⇒ ¬p

- ***At least 10 inches of rain fell today in Miami***
  - It is not the case that at least 10 inches of rain fell today in Miami
  - Less than 10 inches of rain fell today in Miami

Exercise Sets 1.5, 1.6, 1.7

I. Form the negation of each of the following statement.

1. It is raining.
2. It is snowing.
3. The Dallas Cowboys are not the team with the most Super Bowl wins.
4. The New York Yankees are not the team with the most World Series wins.
5. It is not true that chocolate in moderation is good for the heart.
6. It is not true that Albert Einstein was offered the presidency of Israel.

Exercise Sets 1.5, 1.6, 1.7

II. Give a negation and the resulting truth value of each of the following inequalities ($x = 1, 11$ & $y = 4, -5$ & $q = 2, 5$ & $r = 20, 17$)

1. $x > 12$.
2. $y < -6$.
3. $q \geq 5$.
4. $r \leq 19$
5. $8 > 11$

III. For each of the following statements:

p: Listening to classical music makes infants smarter.
q: Subliminal advertising makes you buy things.
r: Sigmund Freud's father was not 20 years older than his mother.
s: Humans and bananas do not share approximately 60% of the same DNA structure.

Find: 1. $\neg p$ 2. $\neg q$ 3. $\neg r$ 4. $\neg s$

IV. Explain why the negation of "$r > 4$" is not "$r < 4$".

1.8 AND Statement "Conjunction"

الوصل - الاقتران

If p and q represent two simple statements then the compound statement "p and q" is symbolized by $p \wedge q$ (P AND q, p conjoined with q). The compound statement formed by connecting statements with the word and is called a conjunction.

The conjunction is :

i. Idempotent: $p \wedge p$ is equivalent to p .

ii. Binary Operator مؤثر ثنائي (takes two propositions.

iii. Commutative تبادلي: $p \wedge q$ is the same as $q \wedge p$.

iv. Associative ترابطي: $p \wedge (q \wedge r)$, is the as $(p \wedge q) \wedge r$, is the same as $r \wedge (q \wedge p)$.

TABLE 2 The Truth Table for the Conjunction of Two Propositions.

p	q	$p \wedge q$
T	T	T
T	F	F
F	T	F
F	F	F

Conjunction: $p \wedge q$ is true when both p and q are true. False otherwise

1.8 AND Statement "Conjunction"

الوصل - الاقتران

- **p**: "Today is Friday", **q**: "It is raining today".

- **p ∧ q** : "Today is Friday AND / BUT لكن / YETمع ذلك / NEVERTHLESS بالرغم من ذلك it is raining today"

  - True: on rainy Fridays.

  - False: otherwise:
    - Any day that is not a Friday
    - Fridays when it does not rain

1.8 AND Statement "Conjunction"
الوصل - الاقتران

Let p and q represent the following simple statements:

p: It is after 5 P.M. q: They are working.

Write each compound statement below in symbolic form:

a. It is after 5 P.M. but they are working.

b. It is not after 5 P.M. yet they are not working.

SO LUTION

a. $p \wedge q$. b. $\neg p \wedge \neg q$.

1.8 AND Statement "Conjunction"

الوصل – الاقتران

Variety of ways to express the conjunction that appear in compound statement

Symbolic Statement	English Statement	P: It is after 5 P.M. q: They a e working
$p \wedge q$	p AND q.	It is after 5 P.M. and they are working.
$p \wedge q$	p BUT q.	It is after 5 P.M. but they are working.
$p \wedge q$	p YET q.	It is after 5 P.M. yet they are working.
$p \wedge q$	p NEVERTHELESS q.	It is after 5 P.M. nevertheless they are working.

Exercise Set 1.8

Exercises 1-6, let p and q represent the following simple statements: p: I'm leaving & q: You're staying. Write each of the following compound statements in symbolic form.

1. I'm leaving and you're staying.
2. You're staying yet I'm leaving.
3. You're staying but I'm not leaving.
4. I'm leaving yet you're not staying.
5. You're not staying, but I'm leaving.
6. I'm not leaving, but you're staying.

1.9 OR Statement "Disjunction"

الفصل / الفسخ / الانفصال

If p and q represent two simple statements then the compound statement "p or q" is symbolized by $p \vee q$ (P OR q, p disjoined with q). The compound statement formed by connecting statements with the word and is called a disjunction.

The disjunction is :

i. Idempotent: $p \vee p$ is equivalent to p .

ii. Binary Operator مؤثر ثنائي (takes two propositions.

iii. Commutative تبادلي: $p \vee q$ is the same as $q \vee p$.

iv. Associative ترابطي: $p \vee (q \vee r)$, is the as $(p \vee q) \vee r$, is the same as $r \vee (q \vee p)$.

TABLE 3 The Truth Table for the Disjunction of Two Propositions.

p	q	$p \vee q$
T	T	T
T	F	T
F	T	T
F	F	F

N.B.

1. Disjunction: p v q is false when both p and q are false. True otherwise.
2. $p \vee$ True is always True.
3. $p \vee$ False is always p.

1.9 OR Statement "Disjunction" الفصل

The connective or can mean two different things. For example, consider this statement: I visited London or Paris. The statement can mean:

i. I visited London or Paris, but not both.

This is an example of the EXCLUSIVE OR, which means "one or the other, but not both."

ii. By contrast, the statement can mean I visited London or Paris or both. This is an example of the INCLUSIVE OR, which means "either or both."

In our study and in mathematics in general, when the connective OR appears, it means the inclusive OR. If p and q represent two simple statements, then the compound statement "p OR q" means p OR q or both. The compound statement formed by connecting statements with the word OR is called a disjunction. The symbol for or is $\vee$. Thus, we can symbolize the compound statement "p or q or both" by $p \vee q$.

1.9 OR Statement "Disjunction"
الفصل

- $p \vee q$: "Today is Friday or it is raining today"
  - True:
    - Today is Friday
    - It is raining today
    - It is a rainy Friday
  - False
    - Today is not Friday and it does not rain

Exercise Set 1.9

In Exercises 1-4, write each symbolic statement in words. Let p and q represent the following simple statements:

p: I study.   q: I pass the course.

1. I study or I pass the course.
2. I pass the course or I do not study.
3. I study or I do not pass the course.
4. I do not study or I do not pass the course.

1.10 Relations Between "Negation", "Conjunction" and "Disjunction"

Transformation Between "Conjunction" and "Disjunction"

- $\neg (p \wedge q) = \neg p \vee \neg q$
- $\neg (p \vee q) = \neg p \wedge \neg q$

These are called De Morgan's Laws

Exercise Sets 1.5 → 1.10

I. In Exercises 1-12, write the negation of each statement.

1. I'm going to Seattle or San Francisco.
2. This course covers logic or statistics.
3. I study or I do not pass.
4. I give up tobacco or I am not healthy.
5. I am not going and he is going'
6. I do not apply myself and I succeed'
7. A bill becomes law and it does not receive majority approval.
8. They see the show and they do not have tickets.
9. $p \vee \neg q$
10. $\neg p \vee q$
11. $p \wedge (q \vee r)$
12. $p \vee (q \wedge r)$

Exercise Sets 1.5 → 1.10

II. *Let p represent the statement* "Chris collects videotapes" *and let q represent the statement* "Jack plays the tuba." *Convert each of the following compound statements into symbols.*

1. Chris collects videotapes and Jack does not play the tuba.
2. Chris does not collect videotapes or Jack does not play the tuba.
3. Chris does not collect videotapes or Jack plays the tuba.
4. Jack plays the tuba and Chris does not collect videotapes.
5. Neither Chris collects videotapes nor Jack plays the tuba.
6. Either Jack plays the tuba or Chris collects videotapes, and it is not the case that both Jack plays the tuba and Chris collects videotapes.

Exercise Sets 1.5 → 1.10

III. Answer the following:

1. If q is false, what must be the truth value of $(p \wedge \sim q) \wedge q$?
2. If q is true, what must be the truth value of $q \vee (q \wedge \sim p)$?
3. If $p \wedge q$ is true, and p is true, then q must be ____.
4. If $p \vee q$ is false, and p is false, then q must be ____.
5. If $\sim(p \vee q)$ is true, what must be the truth values of the component statements?
6. If $\sim(p \wedge q)$ is false, what must be the truth values of the component statements?

IV. Let p, q, and r represent true, false, and false statements respectively. Find the truth value of the given compound statement:

1. $(p \wedge r) \vee \sim q$
2. $(\sim p \wedge q) \vee \sim r$
3. $\sim[(\sim p \wedge q) \vee r]$
4. $(q \vee \sim r) \wedge p$
5. $\sim(p \wedge q) \wedge (r \vee \sim q)$
6. $\sim[r \vee (\sim q \wedge \sim p)]$
7. $p \wedge (q \vee r)$
8. $(\sim r \wedge \sim q) \vee (\sim r \wedge q)$

Exercise Sets 1.5 → 1.10

V. Give the number of rows in the truth table of the following statements.

1. $p \vee \sim r$
2. $(\sim p \wedge q) \vee (\sim r \vee \sim s) \wedge r$
3. $[(\sim p \wedge \sim q) \wedge (\sim r \wedge s \wedge \sim t)] \wedge (\sim u \vee \sim v)$
4. $p \wedge (r \wedge \sim s)$
5. $[(p \vee q) \wedge (r \wedge s)] \wedge (t \vee \sim p)$
6. $[(\sim p \wedge \sim q) \vee (\sim r \vee \sim s)] \vee [(\sim m \wedge \sim n) \wedge (u \wedge \sim v)]$

VI. If the truth table for a certain compound statement has 64 rows, how many distinct component statements does it have?

VII. Is it possible for the truth table of a compound statement to have exactly 48 rows? Why or why not?

Exercise Sets 1.5 → 1.10

VIII. Construct the truth table for:

1. $\sim q \wedge (\sim p \vee q)$
2. $(\sim p \wedge \sim q) \vee (\sim p \vee q)$
3. $(\sim p \wedge \sim q) \vee (\sim r \vee \sim p)$
4. $\sim(\sim p \wedge \sim q) \vee (\sim r \vee \sim s)$
5. $\sim p \vee (\sim q \wedge \sim p)$
6. $(\sim p \wedge q) \wedge r$
7. $(p \vee \sim q) \wedge (p \wedge q)$
8. $r \vee (p \wedge \sim q)$
9. $(\sim r \vee \sim p) \wedge (\sim p \vee \sim q)$
10. $(\sim r \vee s) \wedge (\sim p \wedge q)$

IX. True or False:

1. For every real number y, $y < 13$ or $y > 6$.
2. For every real number t, $t > 9$ or $t < 9$.
3. For some integer p, $p \geq 4$ and $p \leq 4$.
4. There exists an integer n such that $n > 0$ and $n < 0$.

X. In Exercises 1-10, write each symbolic statement in words. Let p and q represent the following simple statements:

p: The father loves his son. q: The son loves his father.

1. $\neg(p \wedge q)$
2. $\neg(q \wedge p)$
3. $\neg p \wedge q$
4. $\neg q \wedge p$
5. $\neg(q \vee p)$
6. $\neg(p \vee q)$
7. $\neg q \vee p$
8. $\neg p \vee q$
9. $\neg p \wedge \neg q$
10. $\neg q \wedge \neg p$

Exercise Sets 1.5 → 1.10

XI. Use one of De Morgan's Laws to write the negation of each of the following statements.

1. For every real number y, $y < 13$ or $y > 6$.
2. For some integer p, $p \geq 4$ and $p \leq 4$.
3. Complete the truth table for *exclusive disjunction*. The symbol $\underline{\vee}$ represents "one or the other is true, but not both."
4. For every real number t, $t > 9$ or $t < 9$.
5. There exists an integer n such that $n > 0$ and $n < 0$.
6. Attorneys sometimes use the phase "and/or." This phrase corresponds to which usage of the word *or*: inclusive or exclusive?

Exercise Sets 1.5 → 1.10

XII. In the following Exercises, choose the correct statement.

The City Council of a large northern metropolis promised its citizens that in the event of snow, all major roads connecting the city to its airport would remain open. The City Council did not keep its promise during the first blizzard of the season. Therefore, during the first blizzard:

a. No major roads connecting the city to the airport were open.

b. At least one major road connecting the city to the airport was not open.

c. At least one major road connecting the city to the airport was open

d. The airport was forced to close.

Exercise Sets 1.5 → 1.10

XIII. In the following Exercises, choose the correct statement.

During the Watergate sandal in 1974, President Richard Nixon assured the American people that "In all my years of public service I, have never obstructed justice. "Later, events indicated that the president was not telling the truth. Therefore in his years of public service:

a. Nixon always obstructed justice.

b. Nixon sometimes did not obstruct justice.

c. Nixon sometimes obstructed justice.

d. Nixon never obstructed justice.

1.11 Expressing Symbolic Statements With Parenthesis In English

Many compound statements contain more than one connective. When expressed symbolically parentheses are used to indicate which simple statements are grouped together. When expressed in English, commas are used to indicate the grouping. Here is a table that illustrates groupings using parentheses in symbolic statements and commas in English statements:

Symbolic Statement	Statements to Group Together	English Translation
$(q \wedge \sim p) \rightarrow \sim r$	$q \wedge \sim p$	If q and not p, then not r.
$q \wedge (\sim p \rightarrow \sim r)$	$\sim p \rightarrow \sim r$	q, and if not p then not r.

1.11 Expressing Symbolic Statements With Parenthesis In English

Symbolic Statement	Statements to Group Together	English Translation
$(q \wedge \sim p) \rightarrow \sim r$	$q \wedge \sim p$	If q and not p, then not r.
$q \wedge (\sim p \rightarrow \sim r)$	$\sim p \rightarrow \sim r$	q, and if not p then not r.

The statement in the first row is an if-then conditional statement. Notice that the symbol $\rightarrow$ is outside the parentheses. By contrast, the statement in the second row is and conjunction. In this case, the symbol $\wedge$ is outside the parentheses. Notice that when we translate the symbolic statement into English, the simple statements in parentheses appear on the same side of the comma.

1.11 Expressing Symbolic Statements With Parenthesis In English

Let p, q, and r represent he following simple statements:

p: A student missed lecture.

q: A student studies.

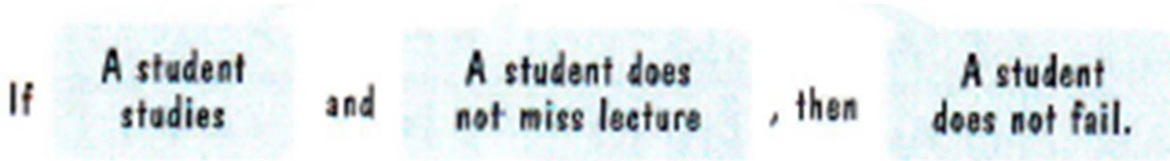
r: A student fails.

Write each of the symbolic statements below in words:

a. $(q \wedge p) \rightarrow \neg r$ b. $q \wedge (\neg p \rightarrow \neg r)$.

Solution

a. $(q \wedge p) \rightarrow \neg r$



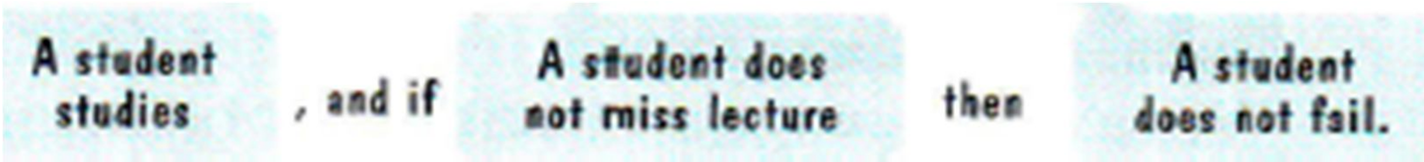
One possible English translation for the symbolic statement is:

If a student studies and does not miss lecture, then the student does not fail.

Observe how the symbolic statements in parentheses appear on the same side of the comma in the English translation.

1.11 Expressing Symbolic Statements With Parenthesis In English

b. $q \wedge (\neg p \rightarrow \neg r)$



One possible English translation for the symbolic statement is:

A student studies, and if a student does not miss lecture then the student does not fail.

Once again, the symbolic statements in parentheses appear on the same side of the comma in the English statement.