Kingdom of Saudi Arabia
Ministry of Higher Education
Majmaah University Faculty of Science-Zulfi
Department of Computer Science & Information

جامعة المجمعة
Majmaah University

Advanced Mathematics For CS

"Discrete Mathematics"

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Lecture Three

1.14 Biconditional Statement

Biconditional Statement: "p if and only if q", has the truth table

TABLE 6 **The Truth Table for the Biconditional $p \leftrightarrow q$.**

p	q	$p \leftrightarrow q$
T	T	T
T	F	F
F	T	F
F	F	T

- $p \leftrightarrow q$ is True when p, q have the same truth value, False otherwise. It is interpreted as the conjunction of two conditionals $p \rightarrow q$ and $q \rightarrow p$. Using symbols this conjunction is written as $(p \rightarrow q) \wedge (q \rightarrow p)$
- Also known as bi-implications

1.14 Biconditional Statement

Example 1. :

- p: “you can take the flight”.
- q: “you buy a ticket”.

- $P \leftrightarrow q$: “You can take the flight if and only if you buy a ticket”
  - This statement is true
    - If you buy a ticket and take the flight
    - If you do not buy a ticket and you cannot take the flight

Exercise Set 1.14.A

Identify each statement as true or false.

1. $5 = 9 - 4$ if and only if $8 + 2 = 10$.
2. $3 + 1 \neq 6$ if and only if $8 \neq 8$.
3. $8 + 7 \neq 15$ if and only if $3 \times 5 \neq 9$.
4. $6 \times 2 = 14$ if and only if $9 + 7 \neq 16$.
5. Bill Clinton was president if and only if Jimmy Carter was not president.
6. Burger King sells Big Macs if and only if IBM manufactures computers.

1.14 Biconditional Statement

Example 2. :

Symbolic Statement	Statements in Words	English Statement
$p \leftrightarrow q$	p if and only if q	A person is an unmarried male if and only if that person is a bachelor.
$p \leftrightarrow q$	q if and only if p	A person is a bachelor if and only if that person is an unmarried male.
$p \leftrightarrow q$	if p then q, and if q then p	If a person is an unmarried male then that person is a bachelor, and if a person is a bachelor then that person is an un married male.
$p \leftrightarrow q$	p is necessary and sufficient for q	Being an unmarried male is necessary and sufficient for being a bachelor.
$p \leftrightarrow q$	q is necessary and sufficient for p	Being a bachelor is necessary and sufficient for being an unmarried male.

Exercise Set 1.14.B

1. In Exercises 1-6, write each compound statement in symbolic form. Let p and q represent the following simple statements:

p: The campus is closed.   q: It is Sunday.

1. The campus is closed if and only if it is Sunday.
2. It is Sunday if and only if the campus is closed.
3. It is not Sunday if and only if the campus is not closed.
4. The campus is not closed if and only if it is not Sunday.
5. Being Sunday is necessary and sufficient for the campus being closed.
6. The campus being closed is necessary and sufficient for being Sunday.

Exercise Set 1.14.B

2. In Exercises 1-8, let p and q represent the following simple statements: p: The heater is working. q: The house is cold. Write each symbolic statement in words.

1. $\neg p \wedge q$	2. $p \wedge q$	3. $p \vee \neg q$
4. $\neg p \vee q$	5. $p \rightarrow \neg q$	6. $q \rightarrow \neg p$
7. $q \leftrightarrow \neg p$	8. $\neg p \leftrightarrow q$	

3. In Exercises 1-8, let p and q represent the following simple statements: q: It is July 4th. r: We are having a barbecue. Write each symbolic statement in words.

1. $q \wedge \neg r$	2. $\neg q \wedge r$	3. $\neg q \vee r$
4. $q \vee \neg r$	5. $r \rightarrow \neg q$	6. $q \rightarrow \neg r$
7. $\neg q \leftrightarrow r$	8. $q \leftrightarrow \neg r$	

4. Explain Why the statement "A week has eight days if and only if December has forty days" is true.

Exercise Set 1.14.B

5. In the following exercises write each compound statement in symbolic form. Let letters assigned to the simple statements represent English sentence that are not negated If commas do not appear in compound English statements use the dominance of connectives to show grouping symbols(parentheses) in symbolic statements.

1. If I like the teacher or the course is interesting then I do not miss class.
2. If the lines so down or the transformer blows then we do not have power.
3. I like the teacher, or if the course is interesting then I do not miss class.
4. The lines go down, or if the transformer blows then we do not have power.
5. I miss class if and only if it's not true that both I like the teacher and the course is interesting.
6. We have power if and only if it's not true that both the lines go down and the transformer blows.
7. If I like the teacher I do not miss class if and only if the course is interesting.
8. If the lines go down we do not have power if and only if the transformer blows.
9. If I do not like the teacher and I miss class then the course is not interesting or I spend extra time reading the textbook.
10. If the lines do not go down and we have power then the transformer does not blow or there is an increase in the cost of electricity.

Exercise Set 1.14.B

6. In the following exercises, write each compound statement in symbolic form using:

p: The temperature outside is freezing.

q: The heater is working.

r: The house is cold.

1. The temperature outside is freezing and the heater is working, or the house is cold.
2. If the temperature outside is freezing, then the heater is working or the house is not cold.
3. If the temperature outside is freezing or the heater is not working, then the house is cold.
4. It is not the case that if the house is cold then the heater is not working.
5. The house is cold, if and only if the temperature outside is freezing and the heater isn't working.
6. If the heater is working, then the temperature outside is freezing if and only if the house is cold.
7. Sufficient conditions for the house being cold are freezing outside temperatures and a heater not working.
8. A freezing outside temperature is both necessary and sufficient for a cold house if the heater is not working.

Exercise Set 1.14.B

7. In the following Exercises, write each compound statement in symbolic form. Assign letters to simple statement that are not negated and show grouping symbols in symbolic statements.

1. If it's not true that being French is necessary for being a Parisian then it's not true that being German is necessary for being a Berliner.

2. If it's not true that being English is necessary for being a Londoner then it's not true that being American is necessary for being a New Yorker.

3. Filing an income tax report and a complete statement of earnings is necessary for each tax payer or an authorized tax preparer.

4. Fatling in love with someone in your class or picking someone to hate are sufficient conditions for showing up to vent your emotions and not skipping. (Paraphrased from a student at the University of Georgia)-

5. It is not true that being wealthy is a sufficient condition for being happy and living contentedly.

6. It is not true that being happy and living contentedly are necessary conditions for being wealthy.

Exercise Set 1.14.B

8. The following exercise contain statements made by well-known people. Use letters to represent each non-negated simple statement and rewrite the given compound statement in symbolic form.

1. "If you cannot get rid of the family skeleton, you may as well make it dance." (George Bernard Shaw)

2. "I wouldn't turn out the way I did if I didn't have all the old-fashioned values to rebel against.“ (Madonna)

3. "If you know what you believe then it makes it a lot easier to answer questions, and I can't answer your question.“ (George W. Bush)

4. "If you don't like what you're doing, you can always pick up your needle and move to another groove." (Timothy Leary)

5. "If I were an intellectual, I would be pessimistic about America, but since I'm not an intellectual, I am optimistic about America." (General Lewis B. Hershey, Director of the Selective Service during the Vietnam war) (For simplicity, regard "optimistic" as "not pessimistic.")

6. "You cannot be both a good socializer and a good writer." (Erskine Caldwell)

Exercise Set 1.14.B

9. In Exercises 1-14, let p, q, and r represent the following simple statements:

p: The temperature is above 85°. q: We finished studying.

r: We go to the beach.

Write each symbolic statement in words. If a symbolic statement is given without parentheses place them, as needed, before and after the most dominant connective and then translate into English.

1. $(p \wedge q) \rightarrow r$	2. $(q \wedge r) \rightarrow P$	3. $p \wedge (q \rightarrow r)$
4. $p \wedge (r \rightarrow q)$	5. $\neg r \rightarrow \neg p \vee \neg q$	6. $\neg p \rightarrow q \vee r$
7. $(\neg r \rightarrow \neg q) \vee p$	8. $(\neg p \rightarrow \neg r) \vee q$	9. $r \leftrightarrow p \wedge q$
10. $r \leftrightarrow q \wedge p$	11. $(p \leftrightarrow q) \wedge r$	12. $q \rightarrow (r \leftrightarrow p)$
13. $\neg r \rightarrow \neg(p \wedge q)$	14. $\neg(p \wedge q) \rightarrow \neg r$	

1.15 Logical Equivalence

Equivalent Statements

Equivalent compound statements are made up of the same simple statements and have the same corresponding truth values for all true-false combinations of these simple statements. If a compound statement is true; then its equivalent statement must also be true. Likewise, if a compound statement is false' its equivalent statement must also be false.

Truth tables are used to show that two statements ale equivalent. When translated in to English, equivalencies can be used to gain a better understanding of English statements.

A special symbol, $\equiv$, is used to show that two statements are equivalent. Because $p \vee \neg q$ and $\neg p \rightarrow \neg q$ are equivalent, we can write

$$p \vee \neg q \equiv \neg p \rightarrow \neg q \quad \text{or} \quad \neg p \rightarrow \neg q \equiv p \vee \neg q$$

The double negation of a statement is equivalent to the statement. For example, the statement "It is not true that Ernest Hemingway was not a writer" means the same thing as "Ernest Hemingway was a writer."

1.15 Logical Equivalence

- Generally

The two different propositions P and Q are said to be Logically equivalent if they have the same truth values, no matter what truth values their constituent have.

In other words, suppose the propositions P and Q are made up of the propositions $p_1, p_2, \dots \dots, p_n$. We say that P and Q are logically equivalent and write :

$$P \equiv Q,$$

Provided that , given any truth values of $p_1, p_2, \dots \dots, p_n$ either P and Q are both true, or P and Q are both false.

For example:

- $(\sim A) \vee B \equiv. A \rightarrow B$
- $\overline{A \wedge B} \equiv \overline{A} \vee \overline{B}$
- $\overline{A \vee B} \equiv \overline{A} \wedge \overline{B}$

} De Morgan's Laws

Exercise Set 1.15.A

In Exercises 1 -12, use a truth table to determine whether the two statements are equivalent.

1. $\neg p \rightarrow q,\ q \rightarrow \neg p$. 2. $\neg p \rightarrow q,\ p \rightarrow \neg q$.
3. $(p \rightarrow \neg q) \wedge (\neg q \rightarrow p),\ p \leftrightarrow q$.
4. $(\neg p \rightarrow q) \wedge (q \rightarrow \neg p)\ ,\ \neg p \leftrightarrow q$.
5. $(p \wedge q) \wedge r\ ,\ p \wedge (q \wedge r)$.
6. $(p \vee q) \vee r\ ,\ p \vee (q \vee r)$.
7. $(p \wedge q) \vee r\ ,\ p \wedge (q \vee r)$.
8. $(p \vee q) \wedge r\ ,\ p \vee (q \wedge r\ )$
9. $(p \vee r) \rightarrow \neg q,\ (\neg p \wedge \neg r) \rightarrow q$
10. $(p \wedge \neg r) \rightarrow q,\ (\neg p \vee r) \rightarrow \neg q$
13. $\neg p \rightarrow (q \vee \neg r),\ (r \wedge \neg q) \rightarrow P$
14. $\neg p \rightarrow (\neg q \vee r),\ (\neg r \wedge q) \rightarrow P$

1.15 Logical Equivalence

Using De Morgan's Laws To Transform Statement To Its Conditional One

- $\neg (p \wedge q) \equiv \neg p \vee \neg q \equiv \quad p \rightarrow \neg q$
- $\neg (p \vee q) \equiv \neg p \wedge \neg q \equiv \neg (\neg p \rightarrow q)$

$\therefore \quad p \wedge q \equiv \neg (p \rightarrow \neg q)$

$\therefore \quad p \vee q \equiv \neg \; p \rightarrow \quad q$

Exercise Set 1.15.B

In Exercises 1 - 16, use De Morgan's laws to write a statement that is equivalent to the given statement.

1. It is not true that Australia and China are both islands.
2. It is not true that Florida and California are both peninsulas
3. It is not the case that my high school encouraged creativity and diversity.
4. It is not the case that the course covers logic and dream analysis.
5. It is not the case that Jewish scripture gives a clear indication of a heaven or an afterlife.
6. It is not true that Martin Luther King, Jr. supported violent protest or the Vietnam war.
7. It is not the case that the United States has eradicated poverty or racism.
8. It is not the case that the movie is interesting or entertaining.
9. $\neg(\neg p \wedge q)$
10. $\neg(p \wedge \neg q)$
11. If you attend lecture and study, you succeed.
12. If you suffer from synesthesia you can literally taste music and smell colors.
13. If he does not cook, his wife or child does.
14. If it is Saturday or Sunday, I, do not work.
15 $p \rightarrow (q \vee \neg r)$,
16. $p \rightarrow (\neg q \wedge \neg r)$,

1.15.1 Logical Equivalence of Conditional and Contrapositive

The easiest way to check for logical equivalence is to see if the truth tables of both variants have *identical last columns* :

p	q	$p \to q$

p	q	$\neg q$	$\neg p$	$\neg q \to \neg p$

Q: why does this work given definition of $\Leftrightarrow$?

1.15.1 Logical Equivalence of Conditional and Contrapositive

The easiest way to check for logical equivalence is to see if the truth tables of both variants have *identical last columns*:

p	q	$p \rightarrow q$
T	T	T
T	F	F
F	T	T
F	F	T

p	q	$\neg q$	$\neg p$	$\neg q \rightarrow \neg p$

1.15.1 Logical Equivalence of Conditional and Contrapositive

The easiest way to check for logical equivalence is to see if the truth tables of both variants have *identical last columns*:

p	q	$p \rightarrow q$
T	T	**T**
T	F	**F**
F	T	**T**
F	F	**T**

p	q	$\neg q$	$\neg p$	$\neg q \rightarrow \neg p$
T	T			
T	F			
F	T			
F	F			

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p	q	$p \to q$
T	T	**T**
T	F	**F**
F	T	**T**
F	F	**T**

p	q	$\neg q$	$\neg p$	$\neg q \to \neg p$
T	T	F		
T	F	T		
F	T	F		
F	F	T		

Q: why does this work given definition of $\Leftrightarrow$?

1.15.1 Logical Equivalence of Conditional and Contrapositive

The easiest way to check for logical equivalence is to see if the truth tables of both variants have *identical last columns*:

p	q	$p \rightarrow q$
T	T	**T**
T	F	**F**
F	T	**T**
F	F	**T**

p	q	$\neg q$	$\neg p$	$\neg q \rightarrow \neg p$
T	T	F	F	
T	F	T	F	
F	T	F	T	
F	F	T	T	

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The easiest way to check for logical equivalence is to see if the truth tables of both variants have *identical last columns*:

p	q	$p \rightarrow q$
T	T	**T**
T	F	**F**
F	T	**T**
F	F	**T**

p	q	$\neg q$	$\neg p$	$\neg q \rightarrow \neg p$
T	T	F	F	**T**
T	F	T	F	**F**
F	T	F	T	**T**
F	F	T	T	**T**

1.15.2 Logical Non-Equivalence of Conditional and Converse

The ***converse*** of a logical implication is the reversal of the implication.

i.e. the converse of $p \rightarrow q$ is $q \rightarrow p$.

e.g. : The converse of

"If Donald is a duck then Donald is a bird."

Is

"If Donald is a bird then Donald is a duck."

As we'll see next: $p \rightarrow q$ and $q \rightarrow p$ are not logically equivalent.

1.15.2 Logical Non-Equivalence of Conditional and Converse

p	q	$p \rightarrow q$	$q \rightarrow p$	$(p \rightarrow q) \leftrightarrow (q \rightarrow p)$

1.15.2 Logical Non-Equivalence of Conditional and Converse

p	q	$p \to q$	$q \to p$	$(p \to q) \leftrightarrow (q \to p)$
T	T			
T	F			
F	T			
F	F			

1.15.2 Logical Non-Equivalence of Conditional and Converse

p	q	$p \rightarrow q$	$q \rightarrow p$	$(p \rightarrow q) \leftrightarrow (q \rightarrow p)$
T	T	T		
T	F	F		
F	T	T		
F	F	T		

1.15.2 Logical Non-Equivalence of Conditional and Converse

p	q	$p \rightarrow q$	$q \rightarrow p$	$(p \rightarrow q) \leftrightarrow (q \rightarrow p)$
T	T	T	T	
T	F	F	T	
F	T	T	F	
F	F	T	T	

1.15.2 Logical Non-Equivalence of Conditional and Converse

p	q	$p \rightarrow q$	$q \rightarrow p$	$(p \rightarrow q) \leftrightarrow (q \rightarrow p)$
T	T	T	T	T
T	F	F	T	F
F	T	T	F	F
F	F	T	T	T

Exercise Set 1.15.B

1. Select the statement that is equivalent to “I saw the original King Kong or the 2005 version.”

a. If I did not see the original King Kong, I saw the 2005 version.
b. I saw both the original King-Kong and the 2005 version.
c. If I saw the original King Kong, I did not see the 2005 version.
d. If I saw the 2005 version, I did not see the original King Kong.

2. Select the statement that is equivalent to “Citizen Kane or Howard the Duck appears in a list of greatest U.S. movies.”

a. If Citizen Kane appears in the list of greatest U.S. movies, Howard the Duck does not.
b. If Howard the Duck does not appear in the list of greatest U.S. movies, then Citizen Kane does'
c. Both Citizen Kane and Howard the Duck appear in a list of greatest U.S. movies.
d. If Howard the Duck appears in the list of greatest U.S. movies, Citizen Kane does not.

Exercise Set 1.15.B

3. One of the following statements is NOT equivalent to all the others. Which one is it?

a. r only if s.
b. r implies s.
c. If r, then s.
d. r is necessary for s.

4. Select the statement that is NOT equivalent to "It is not true that Sondheim and Picasso are both musicians."

a. Sondheim is not a musician or Picasso is not a musician.
b. If Sondheim is a musician, then Picasso is not a musician.
c. Sondheim is not a musician and Picasso is not a musician.
d. If Picasso is a musician, then Sondheim is not a musician.

5. Select the statement that is NOT equivalent to "It is not true that England and Africa are both countries."

a. If England is a country, then Africa is not a country.
b. England is not a country and Africa is not a country.
c. England is not a country or Africa is not a country.
d. If Africa is a country, then England is not a country.

Exercise Set 1.15.B

6. Select the statement that is NOT equivalent to :

"Miguel is blushing or sunburned."

a. If Miguel is blushing, then he is not sunburned.

b. Miguel is sun burned or blushing.

c. If Miguel is not blushing, then he is sunburned.

d. If Miguel is not sun burned, then he is blushing.

7. Select the statement that is NOT equivalent to :

"If it's raining, then I need a jacket."

a. It's not raining or I need a jacket.

b. I need a jacket or it's not raining.

c. If I need a jacket, then it's raining.

d. If I do not need a jacket, then it's not raining.

Exercise Set 1.15.B

8. In Exercises 1 - 8, determine which, if any, of the three given three statements are equivalent. You may use information about a conditional statement's converse, inverse, or contrapositive, De Morgan's laws, or truth tables.

1. a. If he is guilty, then he does not take a lie-detector test.
   b. He is not guilty or he takes a lie-detector test.
   c. If he is not guilty, then he takes a lie-detector test.

2. a. If the train is late, then I am not in class on time.
   b. The train is late or I am in class on time.
   c. If I am in class on time, then the train is not late.

3. a. It is not true that I have a ticket and cannot go.
   b. I do not have a ticket and can go.
   c. I have a ticket or I cannot go.

Exercise Set 1.15.B

4. a. I work hard or I do not succeed.
   b. It is not true that I do not work hard and succeed.
   c. I do not work hard and I do succeed.

5. a. If the grass turns yellow, you did not use fertilizer or water.
   b. If you use fertilizer and water, the grass will not turn yellow.
   c. If the grass does not turn yellow, you used fertilizer and water.

6. a. If you do not file or provide fraudulent information, you will be prosecuted.
   b. If you file and do not provide fraudulent information, you will not be prosecuted.
   c. If you are not prosecuted, you filed or did not provide fraudulent information.

Exercise Set 1.15.B

7. a. I'm leaving, and Tom is relieved or Sue is relieved.

   b. I'm leaving, and it is false that Tom and Sue are not relieved.

   c. If I'm leaving, then Tom is relieved or Sue is relieved.

8. a. You play at least three instruments, and if you have a master's degree in music then you are eligible.

   b. You are eligible, if and only if you have a master's degree in music and play at least three instruments.

   c. You play at least three instruments, and if you are not eligible then you do not have a master's degree in music.

1.16 Tautology and Contradiction

Tautology بطبيعتها صادقة:

If all the outputs of a compound statement is True, hence that statement is Tautology منتهى الثقة.

Self-Contradiction بطبيعتها كاذبة:

If all the outputs of a compound statement is False, hence that statement is self-contradiction تعارض ذاتي

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Exercise Set 1.16.A

In Exercises, use a truth table to determine whether each statement is a tautology, a s elf-contradiction or, neither.

1. $[(p \rightarrow q) \wedge q] \rightarrow p$
2. $[(p \rightarrow q) \wedge p] \rightarrow q$
3. $[(p \rightarrow q) \wedge \sim q] \rightarrow \sim p$
4. $[(p \rightarrow q) \wedge \sim p] \rightarrow \sim q$
5. $[(p \vee q) \wedge p] \rightarrow \sim q$
6. $[(p \vee q) \wedge \sim q] \rightarrow p$
7. $(p \rightarrow q) \rightarrow (\sim p \vee q)$
8. $(q \rightarrow p) \rightarrow (p \vee \sim q)$
9. $(p \wedge q) \wedge (\sim p \vee \sim q)$
10. $(p \vee q) \wedge (\sim p \wedge \sim q)$
11. $\sim(p \wedge q) \leftrightarrow (\sim p \wedge \sim q)$
12. $\sim(p \vee q) \leftrightarrow (\sim p \wedge \sim q)$
13. $(p \rightarrow q) \leftrightarrow (q \rightarrow p)$
14. $(p \rightarrow q) \leftrightarrow (\sim p \rightarrow \sim q)$
15. $(p \rightarrow q) \leftrightarrow (\sim p \vee q)$
16. $(p \rightarrow q) \leftrightarrow (p \vee \sim q)$
17. $(p \leftrightarrow q) \leftrightarrow [(q \rightarrow p) \wedge (p \rightarrow q)]$
18. $(q \leftrightarrow p) \leftrightarrow [(p \rightarrow q) \wedge (q \rightarrow p)]$
19. $(p \wedge q) \leftrightarrow (\sim p \vee r)$
20. $(p \wedge q) \rightarrow (\sim q \vee r)$
21. $[(p \rightarrow q) \wedge (q \rightarrow r)] \rightarrow (p \rightarrow r)$
22. $[(p \rightarrow q) \wedge (q \rightarrow r)] \rightarrow (\sim r \rightarrow \sim p)$
23. $[(q \rightarrow r) \wedge (r \rightarrow \sim p)] \leftrightarrow (q \wedge p)$
24. $[(q \rightarrow \sim r) \wedge (\sim r \rightarrow p)] \leftrightarrow (q \wedge \sim p)$

Exercise Set 1.16.B

Two statements that can both be true about the same object are **consistent.** *For example,* "It is brown" *and* "It weighs 50 pounds" *are consistent statements. Statements that cannot both be true about the same object are called* **contrary;** "It is a Dodge" *and* "It is a Toyota" *are contrary.*

In the following exercises label each pair of statements as either contrary or consistent

1. Elvis is alive. Elvis is dead.
2. George W. Bush is a Democrat. George W. Bush is a Republican.
3. That animal has four legs. That animal is a dog.
4. That book is nonfiction. That book costs more than $70.
5. This number is an integer. This number is irrational.
6. This number is positive. This number is a natural number.
7. Make up two statements that are consistent.
8. Make up two statements that are contrary.

1.17 Precedence & Dominance of Logical Operators
أولوية و هيمنة الروابط المنطقية

- Precedence/Priority: الأولوية/الأسبقية (في التنفيذ)

$$\neg,\ \wedge,\ \vee,\ \rightarrow,\ \leftrightarrow.$$

- Dominance: الهيمنة/السيطرة (التأثير و الترتيب)

$$\leftrightarrow\ ,\ \rightarrow\ ,\ \vee\ ,\ \wedge\ ,\ \neg.$$

1.17 Precedence & Dominance of Logical Operators → Illustrative Example

- Show how the following expression is executed:

$$\neg p \wedge q \vee r \rightarrow s \vee r \leftrightarrow t \leftrightarrow u \rightarrow w \rightarrow x \leftrightarrow y \rightarrow z$$

Firstly: We have to Rearrange (establish the dominance) this expression

1. Determine the Most Dominant Operator and its both wings حدد المؤثر الأكثر هيمنة

$$\{\neg p \wedge q \vee r \rightarrow s \vee r\} \leftrightarrow \langle t \leftrightarrow u \rightarrow w \rightarrow x \leftrightarrow y \rightarrow z\rangle$$

2.A Enter the LHS

2.B Determine its Most Dominant Operator

$$\{\neg p \wedge q \vee r \rightarrow s \vee r\} \leftrightarrow \langle t \leftrightarrow u \rightarrow w \rightarrow x \leftrightarrow y \rightarrow z\rangle$$

2.C Continue inside the LHS

Don't touch the RHS till you completely finish the LHS

$$\{[\neg p \wedge q \vee r] \rightarrow (s \vee r)\} \leftrightarrow \langle t \leftrightarrow u \rightarrow w \rightarrow x \leftrightarrow y \rightarrow z\rangle$$

2.D Finish the LHS

3. Go to the RHS and Repeat Step 2. For it.

$$\{[(\neg p \wedge q) \vee r] \rightarrow (s \vee r)\} \leftrightarrow \langle t \leftrightarrow \{u \rightarrow w \rightarrow x \leftrightarrow y \rightarrow z\}\rangle$$

Leave the LHS and concentrate on the RHS

Do the same steps you have already made for the LHS.

$$\{[(\neg p \wedge q) \vee r] \rightarrow (s \vee r)\} \leftrightarrow \langle t \leftrightarrow \{[u \rightarrow w \rightarrow x] \leftrightarrow (y \rightarrow z)\}\rangle$$

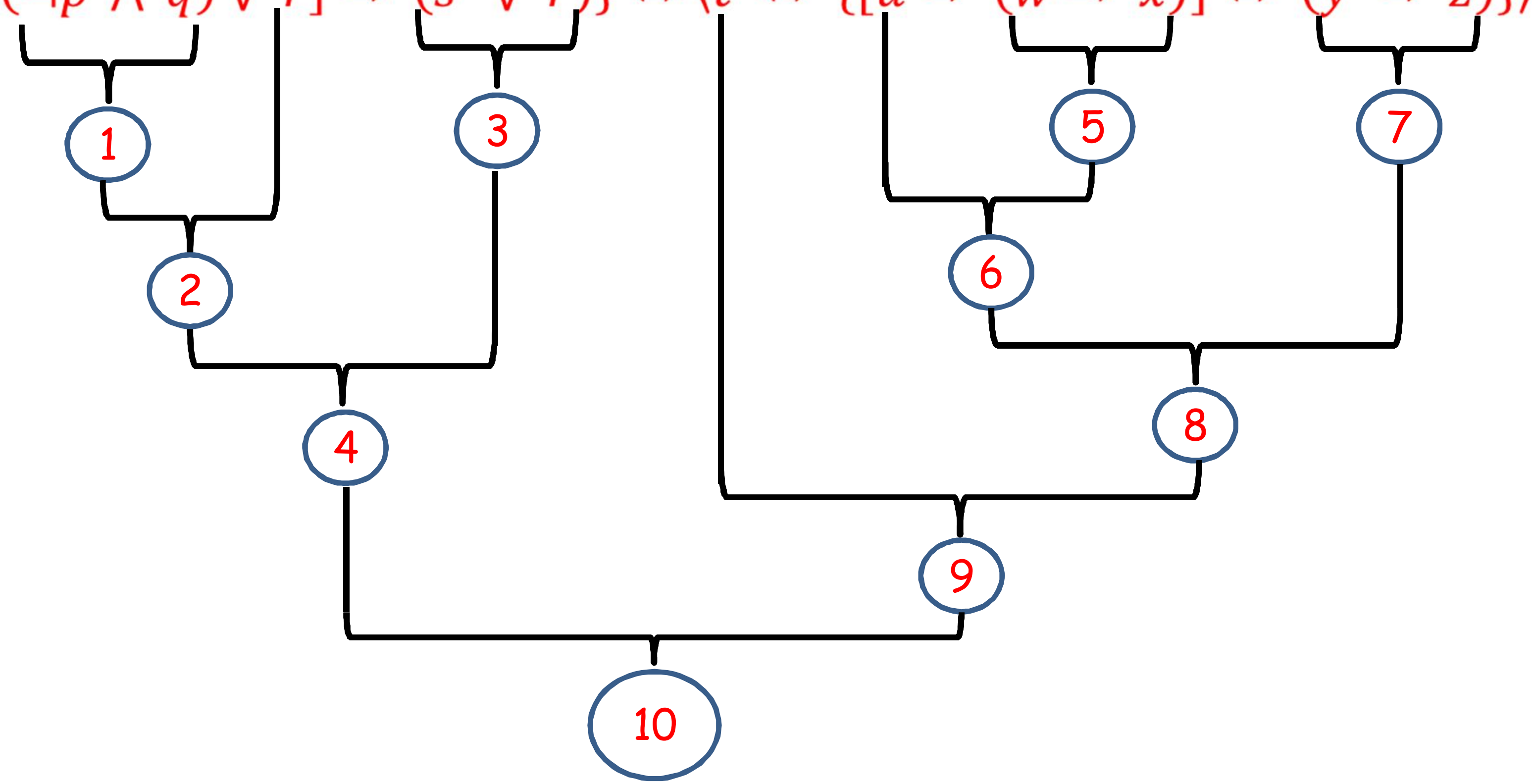
$$\{[(\neg p \wedge q) \vee r] \rightarrow (s \vee r)\} \leftrightarrow \langle t \leftrightarrow \{[u \rightarrow (w \rightarrow x)] \leftrightarrow (y \rightarrow z)\}\rangle$$

1.17 Precedence & Dominance of Logical Operators Illustrative Example

- <u>Secondly</u> :

  we set the priority of operations from left to right

- $\{[(\neg p \land q) \lor r] \rightarrow (s \lor r)\} \leftrightarrow \langle t \leftrightarrow \{[u \rightarrow (w \rightarrow x)] \leftrightarrow (y \rightarrow z)\}\rangle$



Exercise Sets 1.17

In the following exercises, use grouping symbols to clarify the meaning of each statement. Then construct a truth table for the statement

1. $p \rightarrow q \leftrightarrow p \wedge q \rightarrow \sim p$
2. $p \rightarrow \sim q \vee r \leftrightarrow p \wedge r$
3. $q \rightarrow p \leftrightarrow p \vee q \rightarrow \sim p$
4. $\sim p \rightarrow q \wedge r \leftrightarrow p \vee r$
5. $p \rightarrow q \vee r \leftrightarrow p \wedge r$
6. $p \wedge q \rightarrow r \leftrightarrow p \vee r$
7. $p \rightarrow p \leftrightarrow p \wedge p \rightarrow \neg p$
8. $p \rightarrow p \leftrightarrow p \vee p \rightarrow \neg p$

1.18 Quantified Statements
الجمل المنتقاة/جمل مبنية كمياً

- In English, we frequently encounter statements containing the words all, each, every, no (or none, or ne), some, there exists, and at least one (for). These words are called quantifiers جسور/محددات الكمية.
- A statement that contains one of these words is a quantified statement جمل جسرية .
- The words *all, each, every,* and *no (ne*) are called universal كلية quantifiers, while words and phrases such as *some, there exists, and at least one (for)* are called existential وجودي quantifiers.
- Quantifiers are used extensively in mathematics to indicate *how many* cases of a particular situation exist.

1.18 Quantified Statements

- Here are some examples:
  - All poets الشعراء are writers.
  - Some people are bigots متعصبون.
  - No math books have pictures.
  - Some students do not work hard.

1.18.1 Equivalent Ways Of Quantified Statements

- Using our knowledge of the English language we can express each quantified statement in two equivalent ways, that is, in two ways that have exactly the same meaning. These equivalent statements are shown in Table 3.1.

TABLE 3.1 EQUIVALENT WAYS OF EXPRESSING QUANTIFIED STATEMENTS

Statement	An Equivalent Way to Express the Statement	Example (Two Equivalent Quantified Statements)
All *A* are *B*.	There are no *A* that are not *B*.	All poets are writers. There are no poets that are not writers.
Some *A* are *B*.	There exists at least one *A* that is a *B*.	Some people are bigots. At least one person is a bigot.
No *A* are *B*.	All *A* are not *B*.	No math books have pictures. All math books do not have pictures.
Some *A* are not *B*.	Not all *A* are *B*.	Some students do not work hard. Not all students work hard.

1.18.2 Negation Of Quantified Statements Including "ALL"

- Forming the negation of a quantified statement can be a bit tricky. One must be careful when forming the negation of a statement involving quantifiers.

- Suppose we want to negate the statement "All writers are poets." Because this statement is false, its negation must be true. The negation is "Not all writers are poets." This means the same thing as "Some writers are not poets." Notice that the negation is a true statement.

Statement

Negation

1.18.2 Negation Of Quantified Statements Including "ALL"

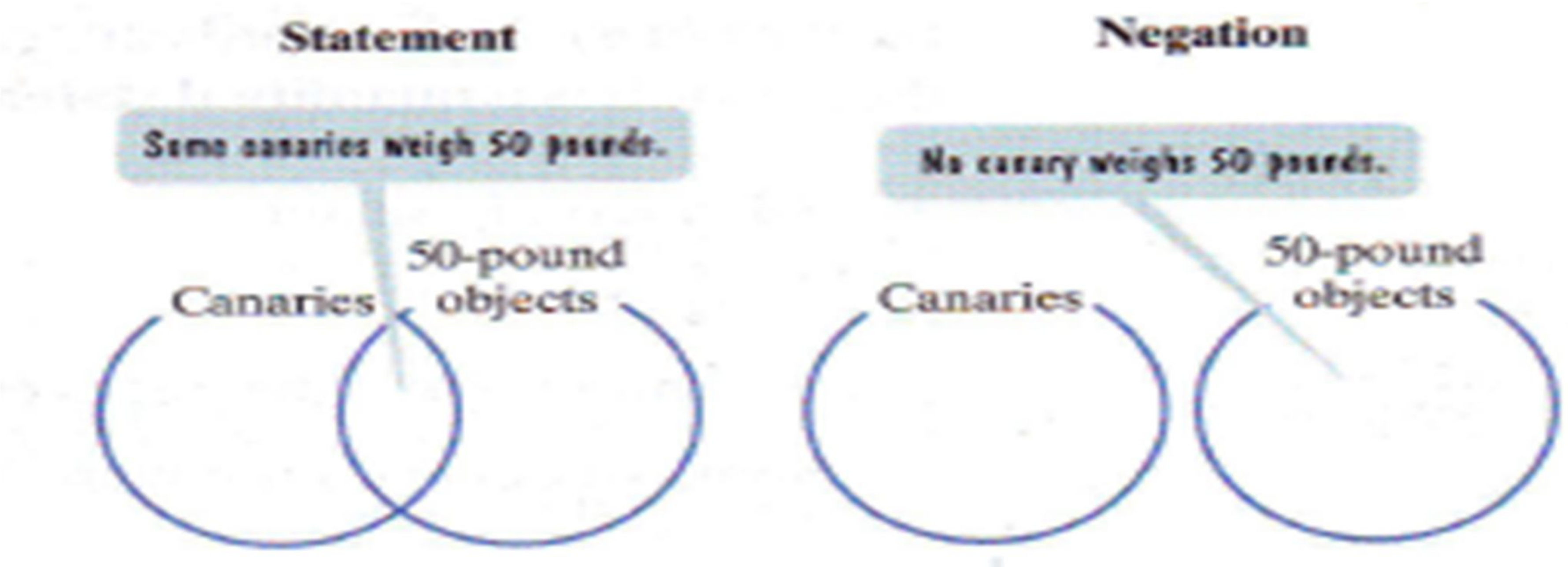
- The negation of "All writers are poets" cannot be "No writers are poets" because both statements are false. The negation of a false statement must be a true statement. In general, the negation of "All A are B" is not "No A are B."

- In general, the negation of "All A are B" is "Some A are not B." Likewise, the negation of "Some A are not B" is "All A ate B."

1.18.2 Negation Of Quantified Statements Including "Some"

- Now let's investigate how to negate a statement with the word some.

- Consider the statement Some canaries طيور الكناري weigh 50 pounds." Because some means" there exists at least one," the negation is "It is not true that there is at least one canary that weighs 50 pounds." Because it is not true that there is even one such critter مخلوق, we can express the negation as "No canary weighs 50 pounds."



1.18.2 Negation Of Quantified Statements

- Illustrative Example 01
- Negate each of the following statements

(a) Some cats have fleas. براغيث

Since *some* means "at least one," the statement "Some cats have fleas" is really the same as "At least one cat has fleas." The negation of this is "No cat has fleas."

(b) Some cats do not have fleas.

This statement claims that at least one cat, somewhere, does not have fleas. The negation of this is "All cats have fleas." تزعم

(c) No cats have fleas.

The negation is "Some cats have fleas."

1.18.2 Negation Of Quantified Statements

- Illustrative Example 02
- Decide whether each of the following statements is *True* or *False*

(a) There exists a whole number that is not a natural number.

Because there is such a whole number (it is 0), this statement is true.

(b) Every integer is a natural number.

This statement is false, because we can find at least one integer that is not a natural number. For example, -1 is an integer but is not a natural number. (There are infinitely many other choices we could have made.)

(c) Every natural number is a rational number.

Since every natural number can be written as a fraction with denominator 1, this statement is true.

(d) There exists an irrational number that is not real. Irrational as: π, $\sqrt{2}$, ……

In order to be an irrational number, a number must first be real (see the box). Therefore, since we cannot give an irrational number that is not real, this statement is false. (Had we been able to find at least one, the statement would have then been true.)

1.18.3 Summary

- Negations of quantified statements are summarized in Table 3.2.

TABLE 3.2 NEGATIONS OF QUANTIFIED STATEMENTS

Statement	Negation	Example (A Quantified Statement and Its Negation)
All *A* are *B*.	Some *A* are not *B*.	All people take exams honestly. The negation is Some people do not take exams honestly.
Some *A* are *B*.	No *A* are *B*.	Some roads are open. The negation is No roads are open.

- (The negations of the statements in the second column are the statements in the first column.)

Statement	Negation
All do.	Some do not. (Equivalently: Not all do.)
Some do.	None do. (Equivalently: All do not.)

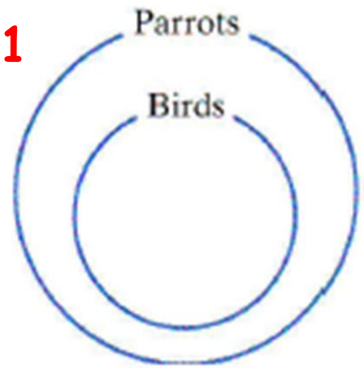
Exercise Set 1.18

I. Exercises 1-4 contain diagrams that show relationships between two sets. (These diagrams are just like the Venn diagrams studied before).

a. Use each diagram to write a statement beginning with the word "all," "some," or "no" that illustrates the relationship between the sets.

b. Determine if the statement in part (a) is true or false. If it is false, write its negation.

1



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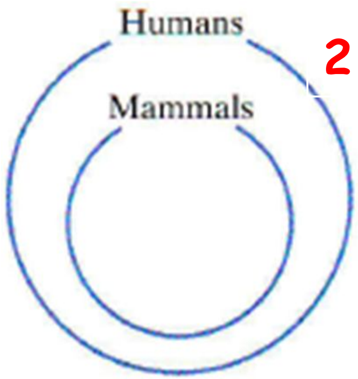

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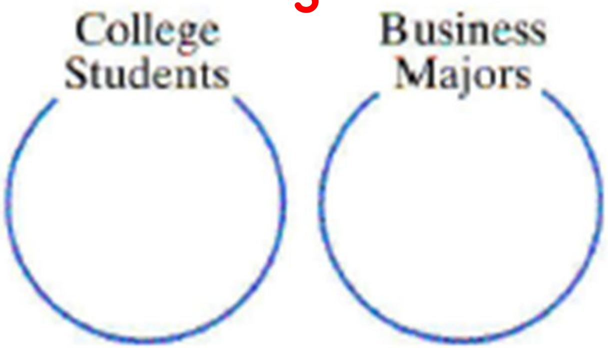

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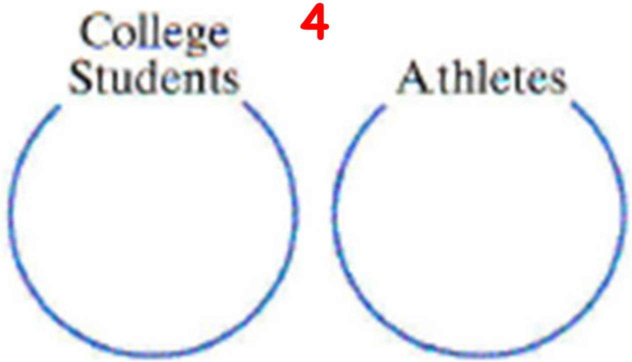


Exercise Set 1.18

II. Exercices 1-8 contain quantifie statements. For each exercise,

a. Express the quantified statement in an equivalent way, that is, in a way that has exactly the same meaning.

b. Write the negation of the quantified statement. (The negation should begin with "all," "some," or "no.")

1. All whales are mammals.
2. All journalists are writers.
3. Some students are business majors.
4. Some movies are comedies.
5. Some thieves are not criminals.
6. Some pianists are not keyboard players.
7. No Democratic presidents have been impeached معزول سياساً.
8. No women have served as Supreme Court justices المحكمة العليا.

Exercise Set 1.18

III. For each of the following statements:

p: No Africans have Jewish ancestry.

q: No religious دينية traditions تقاليد recognize sexuality as central to their understanding of the sacred مقدسات.

r: All rap is hip-hop.

s: Some hip-hop is not rap.

Find:

1. $\neg p$
2. $\neg q$
3. $\neg r$
4. $\neg s$

Exercise Set 1.18

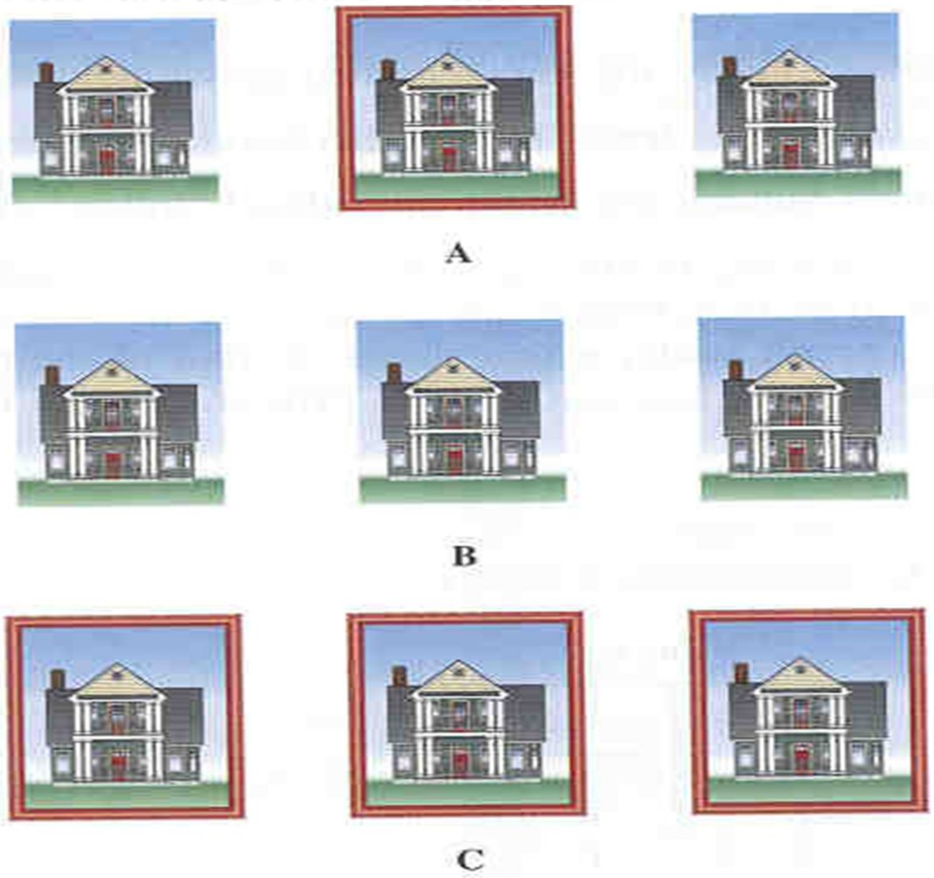
iv. *Write a negation for each of the following statements.*

1. Her aunt's name is Lucia.
2. The flowers are to be watered.
3. Every dog has its day.
4. No rain fell in southern California today.
5. Some books are longer than this book.
6. All students present will get another chance.
7. No computer repairman can play blackjack.
8. Some people have all the luck.
9. Everybody loves somebody sometime.
10. Everyone loves a winner.

Exercise Set 1.18

v. *Refer to the groups of art labeled A, B, and C, and identify by letter the group or groups that are satisfied by the given statements involving quantifiers.*

1. All pictures have frames.
2. No picture has a frame.
3. At least one picture does not have a frame.
4. Not every picture has a frame.
5. At least one picture has a frame.
6. No picture does not have a frame.
7. All pictures do not have frames.
8. Not every picture does not have a frame.



Exercise Set 1.18

vi. *Decide whether each statement in Exercises 65–74 involving a quantifier is* true *or* false.

1. Every whole number is an integer.
2. Every natural number is an integer.
3. There exists a rational number that is not an integer.
4. There exists an integer that is not a natural number.
5. All rational numbers are real numbers.
6. All irrational numbers are real numbers.
7. Some rational numbers are not integers.
8. Some whole numbers are not rational numbers.
9. Each whole number is a positive number.
10. Each rational number is a positive number.

Exercise Set 1.18

vii. Explain the difference between the following statements:

- All students did not pass the test.
- Not all the students passed the test.

viii. Write the following statement using "every":

There is no one here who has not done that at one time or another.

ix. Only one of the following statements is <u>True</u>. Which one is it:

a. For some real number x, $x \nless 0$.

b. For all real numbers x, $x^3 > 0$.

c. For all real numbers x less than 0, x^2 is also less than 0.

d. For some real number x, $x^2 < 0$.

Exercise Set 1.18

X.a. Express each statement in an equivalent way that begins with "all“, "some“, or "no."
X.b. Write the negation of the statement in part (a).

1. Nobody doesn't like Sara Lee.
2. A problem well stated is a problem half solved.
3. Nothing is both safe and exciting.
4. Many a person has lived to regret a misspent youth.
5. Not every great actor is a Tom Hanks.
6. Not every generous philanthropist is a Bill Gates.

Exercise Set 1.18

XI. Read carefully the following paragraph, hence choose the correct statement.

The City Council of a large northern metropolis promised its citizens that in the event of snow, all major roads connecting the city to its airport would remain open. The City Council did not keep its promise during the first blizzard of the season. Therefore, during the first blizzard:

a. No major roads connecting the city to the airport were open.

b. At least one major road connecting the city to the airport was not open.

c. At least one major road connecting the city to the airport was open

d. The airport was forced to close.

Summary of Statements of Symbolic Logic

Name	Symbolic Form	Common English Translations
Negation	$\sim p$	Not p. It is not true that p.
Conjunction	$p \wedge q$	p and q. p but q.
Disjunction	$p \vee q$	p or q.
Conditional	$p \rightarrow q$	If p, then q. p is sufficient for q. q is necessary for p.
Biconditional	$p \leftrightarrow q$	p if and only if q. p is necessary and sufficient for q.

Summary of Statements of Symbolic Logic

VARIATIONS OF THE CONDITIONAL STATEMENT

Name	Symbolic Form	English Translation
Conditional	$p \to q$	If p, then q.
Converse	$q \to p$	If q, then p.
Inverse	$\sim p \to \sim q$	If not p, then not q.
Contrapositive	$\sim q \to \sim p$	If not q, then not p.

THE NEGATION OF A CONDITIONAL STATEMENT

The negation of $p \to q$ is $p \wedge \sim q$. This can be expressed as

$$\sim(p \to q) \equiv p \wedge \sim q.$$

	Symbolic Statement	English Translation
Given Conditional Statement	$p \to \sim q$	If it's a lead pencil, then it does not contain lead. (true)
Converse: Reverse the components of $p \to \sim q$.	$\sim q \to p$	If it does not contain lead, then it's a lead pencil. (not necessarily true)
Inverse: Negate the components of $p \to \sim q$.	$\sim p \to \sim(\sim q)$ simplifies to $\sim p \to q$	If it is not a lead pencil, then it contains lead. (not necessarily true)
Contrapositive: Reverse and negate the components of $p \to \sim q$.	$\sim(\sim q) \to \sim p$ simplifies to $q \to \sim p$	If it contains lead, then it's not a lead pencil. (true)

Summary of Statements of Symbolic Logic

THE CONDITIONAL STATEMENT $p \rightarrow q$

CONTRAPOSITIVE

$p \rightarrow q$ is equivalent to $\sim q \rightarrow \sim p$ (the contrapositive).

CONVERSE AND INVERSE

1. $p \rightarrow q$ is not equivalent to $q \rightarrow p$ (the converse).
2. $p \rightarrow q$ is not equivalent to $\sim p \rightarrow \sim q$ (the inverse).

NEGATION

The negation of $p \rightarrow q$ is $p \wedge \sim q$.

Summary of Statements of Symbolic Logic

De Morgan Law

p	q	$p \wedge q$	$\sim(p \wedge q)$	$\sim p$	$\sim q$	$\sim p \vee \sim q$
T	T	T	F	F	F	F
T	F	F	T	F	T	T
F	T	F	T	T	F	T
F	F	F	T	T	T	T

DE MORGAN'S LAWS AND NEGATIONS

1. $\sim(p \wedge q) \equiv \sim p \vee \sim q$
   The negation of $p \wedge q$ is $\sim p \vee \sim q$. To negate a conjunction, negate each component statement and change *and* to *or*.
2. $\sim(p \vee q) \equiv \sim p \wedge \sim q$
   The negation of $p \vee q$ is $\sim p \wedge \sim q$. To negate a disjunction, negate each component statement and change *or* to *and*.

De Morgan's laws can be used to write the negation of a compound statement that is a conjunction ($\wedge$, and) or disjunction ($\vee$, or).